

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS

9709/02
Paper 2 Pure Mathematics 2 (P2)

| Additional Materials: | Answer Booklet/Paper <br> Graph Paper <br> List of Formulae (MF9) |
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## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet. Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 Solve the inequality $|x-3|>|x+2|$.

2 The variables $x$ and $y$ satisfy the relation $3^{y}=4^{x+2}$.
(i) By taking logarithms, show that the graph of $y$ against $x$ is a straight line. Find the exact value of the gradient of this line.
(ii) Calculate the $x$-coordinate of the point of intersection of this line with the line $y=2 x$, giving your answer correct to 2 decimal places.

3 The parametric equations of a curve are

$$
x=3 t+\ln (t-1), \quad y=t^{2}+1, \quad \text { for } t>1
$$

(i) Express $\frac{\mathrm{d} y}{\mathrm{~d} x}$ in terms of $t$.
(ii) Find the coordinates of the only point on the curve at which the gradient of the curve is equal to 1 .

4 The polynomial $2 x^{3}-3 x^{2}+a x+b$, where $a$ and $b$ are constants, is denoted by $\mathrm{p}(x)$. It is given that $(x-2)$ is a factor of $\mathrm{p}(x)$, and that when $\mathrm{p}(x)$ is divided by $(x+2)$ the remainder is -20 .
(i) Find the values of $a$ and $b$.
(ii) When $a$ and $b$ have these values, find the remainder when $\mathrm{p}(x)$ is divided by $\left(x^{2}-4\right)$.

5 (i) By sketching a suitable pair of graphs, show that the equation

$$
\sec x=3-x
$$

where $x$ is in radians, has only one root in the interval $0<x<\frac{1}{2} \pi$.
(ii) Verify by calculation that this root lies between 1.0 and 1.2.
(iii) Show that this root also satisfies the equation

$$
\begin{equation*}
x=\cos ^{-1}\left(\frac{1}{3-x}\right) \tag{1}
\end{equation*}
$$

(iv) Use the iterative formula

$$
x_{n+1}=\cos ^{-1}\left(\frac{1}{3-x_{n}}\right)
$$

with initial value $x_{1}=1.1$, to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

6 (i) Express $\cos ^{2} x$ in terms of $\cos 2 x$.
(ii) Hence show that

$$
\begin{equation*}
\int_{0}^{\frac{1}{3} \pi} \cos ^{2} x \mathrm{~d} x=\frac{1}{6} \pi+\frac{1}{8} \sqrt{ } 3 \tag{4}
\end{equation*}
$$

(iii) By using an appropriate trigonometrical identity, deduce the exact value of

$$
\int_{0}^{\frac{1}{3} \pi} \sin ^{2} x \mathrm{~d} x
$$



The diagram shows the part of the curve $y=\mathrm{e}^{x} \cos x$ for $0 \leqslant x \leqslant \frac{1}{2} \pi$. The curve meets the $y$-axis at the point $A$. The point $M$ is a maximum point.
(i) Write down the coordinates of $A$.
(ii) Find the $x$-coordinate of $M$.
(iii) Use the trapezium rule with three intervals to estimate the value of

$$
\int_{0}^{\frac{1}{2} \pi} \mathrm{e}^{x} \cos x \mathrm{~d} x
$$

giving your answer correct to 2 decimal places.
(iv) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part (iii).

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